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ABSTRACT

A series of experiments, all concerned with the topic of mathematical concept learning by the young child, were conducted. The concept mainly focused upon was that of inequalities. Five experiments were run. These are: (1) acquisition of mathematical concepts under experimental situation and in traditional pedagogical situation, (2) cued vs. non-cued instructions in the learning of inequalities by three-year-olds, (3) "more than", "less than" and reversal learning, (4) correction vs. non-correction, and (5) effect of pre-training and cued reversal training. The stimuli for the experimental situation were made up of pairs of pictures of varying numbers of objects with from 1 to 3 or 2 to 6 objects in each picture. The experiments provide evidence that nursery school aged children can learn simple mathematical concepts and learn them very rapidly. Children generally found the concept of "more than" more easily understood than the concept of "less than". Problems for further study concern the time to initiate pencil and paper activity, the time to initiate verbal activity, and the question of whether the concept of "more than" is learned as a number concept. (Author/CK)

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MATHEMATICAL CONCEPT LEARNING BY THE PRE-SCHOOL CHILD

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San Jose State College Foundation for and on behalf of
San Jose State College Department of Psychology

San Jose, California

August 1971

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Table of Contents

	Page
List of Tables	
List of Figures	
Introduction	1
Experiment 1	3
Part 1	3
Part 2	7
Discussion	7
Experiment 2	9
Results	10
Discussion	12
Experiment 3	13
Subjects	13
Design	13
Results	14
Experiment 4	17
Design	17
Results	17
Discussion	18
Experiment 4 Follow-up	18
Experiment 5	19
Results and Discussion	20
General Discussion and Some Conclusions	21

List of Tables

Table		Page
1	Experimental design (Experiment 2)	10
2	Number of Ss reaching criterion (Experiment 2) . .	11
3	Mean trials to criterion for the learners (Experiment 2).	11
4	Mean errors to criterion for the learners (Experiment 2).	11
5	Experimental design (Experiment 3)	13
6	Experimental design (Experiment 4)	17
7	Trials to criterion and number of Ss meeting criterion (Experiment 4).	17
8	Experimental design (Experiment 5)	20
9	Trials to criterion (Experiment 5)	20

List of Figures

Figure		Page
1	Examples of stimuli (Experiment 1)	2
2	Mean trials to criterion (Experiment 1).	5
3	Mean number correct on paper and pencil test (Experiment 1)	6
4	Examples of stimuli (Experiment 3)	14
5	Mean trials to criterion on "more than" and "less than" (Experiment 3).	15
6	Mean trials to criterion by groups with pre-training and no pre-training (Experiment 3) . .	16

Introduction

The series of experiments described in this final report are all concerned with the topic of mathematical concept learning by the young child. The concept given particular attention is that of inequalities, a concept important in itself as basic to mathematics and important also because it is one to which the child is commonly introduced at the first grade or even the kindergarten level. Piaget maintains that the understanding of mathematical concepts develops at an age many years beyond that of the children who are the Ss in the present experiments. But it has been pointed out that Piaget's conclusions are drawn from evidence of the child's verbal facility in dealing with mathematical concepts. For many psychologists -and educators- the child's ability to use a mathematical concept (even if he cannot define it nor talk about it correctly and, in particular, to use it in a <u>new</u> situation--transfer) is sufficient evidence that he has acquired the concept. In the experiments summarized in this report, effective transfer is considered acceptable evidence that concept acquisition has been established.

In fact from the evidence presented here, it is clear that the normal three-year-old can, rather quickly, acquire the concept of "more than" and demonstrate such acquisition by successful transfer to an otherwise extremely difficult situation. All the Ss in fact, from 3 to 5½ years old, show little difficulty in acquiring the mathematical concepts presented. The experiments presented here establish such capacity, and also examine reversal shift, the effect of verbal instructions, the advantage of pre-training, certain methodological aspects (overt correction vs. non-correction), and briefly investigate the effectiveness of the pedagogical method in mathematical concept learning of the very young child.

Five rather considerable experiments have been run during the period of the present grant. These are: 1) acquisition of mathematical concepts in the a) experimental situation and b) traditional pedagogical situation, 2) cued vs. non-cued instructions in the learning of inequalities by three-year-olds, 3) "more than", "less than" and reversal learning, 4) correction vs. non-correction, and 5) effect of pre-training and cued reversal training. The experiments have not necessarily been run successively in the numbered order. For example, Experiment 1 -which includes a pedagogical session- was run separately from, but concurrently with, Experiments 2 and 3. Experiments 2, 3, 4 and 5 are an integrated group of experiments in which supportive results are provided across experiments and from which firm conclusions may be drawn.

To clarify the presentation of the experiments, the design, procedure and analysis of each will be presented separately in summarized form (the details have been given in previous Progress Reports) and will be followed by a final integrative discussion.

Stimuli

The stimulus material used in all the experiments was described in detail in the first Progress Report. In brief the stimuli for the experimental situation are made up of pairs of pictures of varying numbers of objects with from 1 to 3 or 2 to 6 objects in each picture. The objects are black line drawings on a white background and -most importantly- may be either "simple" (striated balls) or "complex" (all objects in any one picture are different from each other). An example of both "simple" and "complex" stimuli is presented in Figure 1.

"Simple" Stimuli

"Complex" Stimuli

Figure 1. Examples of pairs of pictures used as stimuli.(1)

(1) A set of stimuli used in any one experimental session is made up of 42 pairs of pictures, so that S is presented with a different pair of pictures on each trial.

Experiment 1

In Experiment 1, nursery school children learn a series of mathematical concepts- one group with and one without pre-training. A third group learns the same concepts by the pedagogical method. All the children are subsequently given the same paper and pencil test.

Part 1

In the first part of the experiment, 45 nursery school children (4-4 to 5-3 years old) in three groups of 15 each were required to learn a series of four number concepts: "more than", "less than", "equipollence" and "non-equipollence". "More than" and "less than" involve separate tasks - the child is presented with a pair of pictures containing different numbers of objects and the child must respond to the picture with "more" or "less" objects- depending upon the task required. On the other hand S acquires both equipollence and non-equipollence via a single task - he is presented with a pair of pictures and must make one response if there are an equal number of objects in both pictures - the alternative response if the number of objects are not equal. Both Group 1 and 2 Ss were taken individually through 42 trials per day for a maximum of two days or to a criterion of eight out of 10 correct responses on each of the three learning tasks. Group 1 Ss learned the concept first with "simple" stimuli and then were transferred to the "complex" stimuli. Group 2 Ss learned the concepts using the "complex" stimuli only. Group 3 Ss were taught the four concepts in a group situation by the traditional pedagogical method and the success of the method was evaluated by the group performance on a paper and pencil test. Group 1 and 2 Ss, after reaching criterion on the four concepts by the experimental method, were given (without any further training) the same paper and pencil test.

Groups 1 and 2. Apparatus. a) Inequalities. Essentially the apparatus was a box with two plexiglass windows on the side facing S. On each trial, E inserted in a slot on the side of the apparatus a pressboard frame containing two pictures, each stamped on a 4-inch square of translucent plastic. Pressure on the window containing the correct picture caused the picture to light up.

b) Equipollence and non-equipollence. The apparatus consisted of a wooden screen with two windows approximately at the young child's eye-level when he was seated on a low chair. One inch beneath each window on S's side was a reinforcement light and one inch below each reinforcement light was a response button. The stimulus (a picture in each window) was presented on each trial by inserting an 8 x 11 inch card. If S made a correct response (pressed the correct button), the reinforcement light above that button went on.

Procedure. When presented on each trial with a pair of pictures for a) "more than" or "less than", the child was required

to press the picture which contained "more" or "less" objects. If he pressed the correct window the picture behind it lit up. If he pressed the incorrect window, nothing happened and he was required to make an overt correction response. When presented with a pair of pictures b) in which there were or there were not an equal number of objects, the child was required to press one of the two response buttons. If there were an equal number of objects in the two pictures, the child pressed one button. If the number was not equal, he pressed the other button. Following a correct button press, the reinforcement light above the button went on. Following an incorrect response, the child was required to make an overt correction response.

Each child in Group 1 received a different randomization of pairs of stimulus pictures and the randomization was matched across the two groups. In the instructions given to the child for each task, the words "more than", "less than", "the same number" or "not the same number" were used, as appropriate. To help the child in making the association between the verbal symbol and the concept, a verbal reinforcement was given every fourth trial. For example, in the inequalities task "more than", E said, following the correct response, "Yes, that picture has more balls". In a different task E might say "Yes, those pictures have the same number of balls - they are equal".

Group 3. The pedagogical method refers to a typical classroom procedure in which the concepts are explained to the children at the outset and are demonstrated with the use of teaching aids, and in which the children are given practice using the concepts with paper and pencil tasks. A "good" credentialled teacher was chosen for this group and Ss were taught in subgroups of eight or seven with the teacher spending six half-hour sessions with each of the two subgroups. The concepts to be taught were explained to the teacher and she was asked to use from one to three objects in each set of objects and to use like and unlike objects in her demonstrations. Otherwise she was given total latitude as to the methods used. She used as teaching aids chalk board, flannel board, work book sheets, crayons and a variety of objects for demonstrations. The teacher also made use of informal work sheets and had the children draw the correct number of objects to demonstrate the concepts, so that Group 3 Ss had practice with pencil and paper tasks. The teacher, finding that it was difficult for a child to learn a concept using unlike objects only, also taught each concept first with like objects, then with unlike objects, thus duplicating the Group 1 experimental situation even though this procedure had not been suggested to her.

Paper and Pencil Task. The final paper and pencil task consisted of an eight page booklet. On the first four pages of the booklet the four concepts were presented in the order they were learned, i.e., "more than", "less than", "equal to" and "not equal to". Two of the examples were presented on each page. On the last four pages the concepts were presented in a different order, also two to a page. All the children, in subgroups of 7 or 8, were

given the paper and pencil test on the same day. The E demonstrated at the blackboard how each page was to be responded to for the first four pages. No demonstration was given for the last four pages. Essentially, the child was asked to look at both pictures on each page, one at a time, and mark the one which demonstrated a particular concept. The paper and pencil task took approximately one half hour for each subgroup.

Results. Groups 1 and 2. All 15 Ss in Group 1 reached criterion on the "more than" and "less than" tasks. But only 13 and 12 learned "equal to" and "not equal to" with the simple and complex stimuli respectively. In Group 2 all 15 Ss learned "more than", but 4 Ss were lost on the "less than" task and 2 more on "equal to" and "not equal to", making six in all who were non-learners. Mean trials to criterion for the two groups are presented graphically in Figure 2.

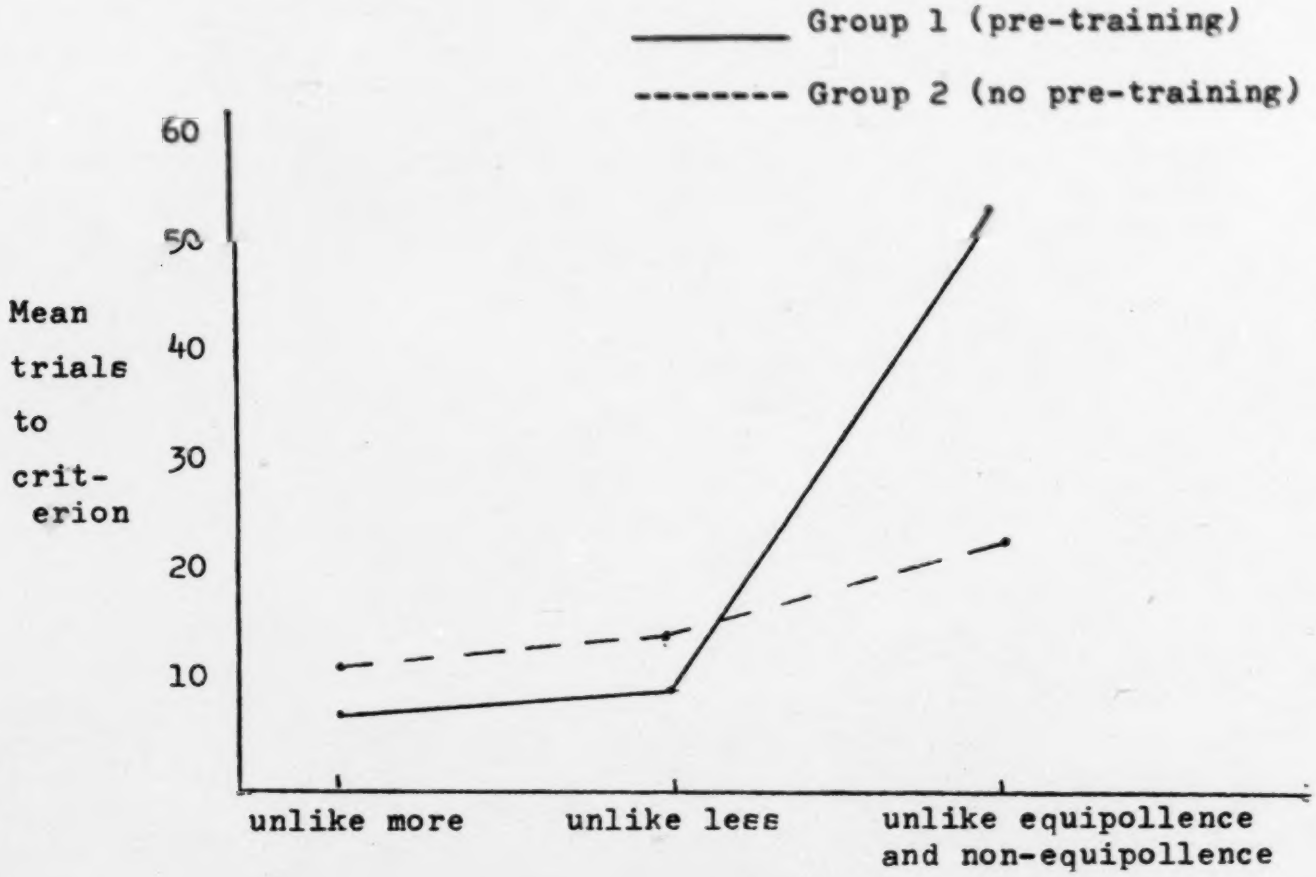

Figure 2. Mean trials to criterion on three concepts by Groups 1 and 2.

It seems clear from Figure 2 that there is little difference between the two groups on the "more than" and "less than" tasks, but a considerable difference on the more difficult task of equipollence and non-equipollence. An analysis of variance performed on trials to criterion on the three learning tasks with the

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complex stimuli only, showed no significant difference between the groups due to pre-training, but a significant task and interaction (task by pre-training)- a result which confirms statistically the graphic presentation of Figure 2. The difference between the mean trials to criterion of the two groups on the third task is highly significant. An examination of the mean errors to criterion on the complex task for the two groups gave much the same results. The difference between the means is not significant for either "more than" or "less than", but for "equipollence" and "non-equipollence" the difference between the mean errors was highly significant.

Test Results. The mean number of correct responses on the three concepts from the paper and pencil test given to the three groups are presented graphically in Figure 3. From Figure 3 it can be seen that there is little difference between the means of the three groups in any of the three concepts and an analysis of variance confirms this observation in that there is no significant difference between the groups.

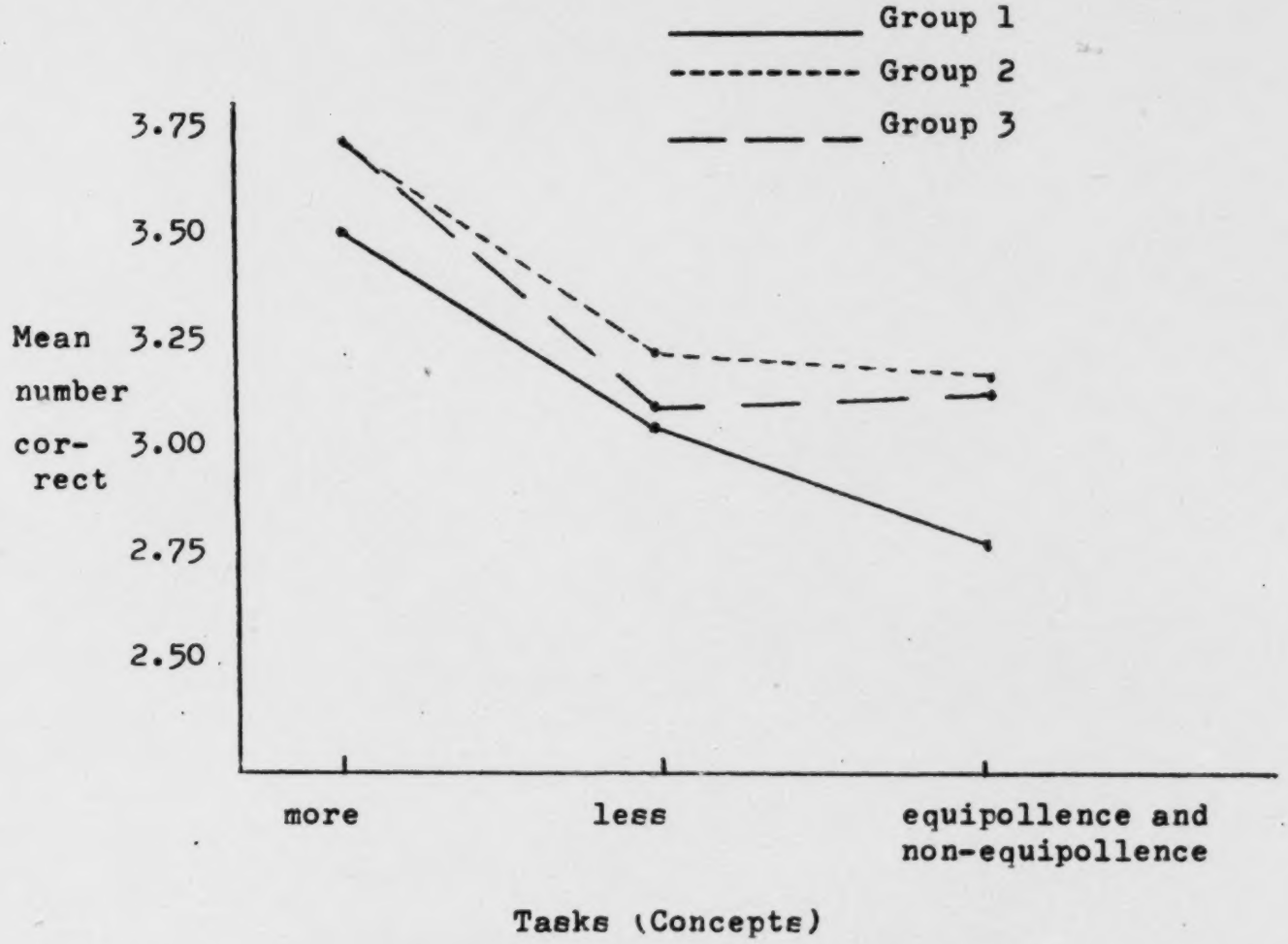

Figure 3. Mean number correct on three concepts from the paper and pencil test by Groups 1, 2 and 3.

Part 2

Although the children in the experimental Groups 1 and 2 did as well on the paper and pencil test as the children trained pedagogically who had paper and pencil practice throughout the pedagogical sessions, nevertheless, the test results may in fact have been a function of competence with a paper and pencil rather then evidence of effective acquisition of the number concepts. Therefore, a further group of children at the same age level were taken through the Group 1 procedure and then given 10 minutes paper and pencil practice with work sheets similar to those used by the teacher in the pedagogical group. Subsequently the same paper and pencil task was given as in part 1. The scores made by the children in this part of the experiment were practically perfect. Seven of the 12 children made a perfect score on all three tasks. No child made a score of less than 14 out of 16 on all three tasks. Clearly the few minutes of paper and pencil training had been highly effective and had enabled all the children to demonstrate acquisition of the experimental concepts.

Discussion

The results of the present experiment indicates unequivocally that children of nursery school age can learn a short series of number concepts which are presented successively. In fact, presentation of the number concepts apparently are not limited to a particular method since successful acquisition followed all three procedures.

"More than" and "less than" were clearly concepts so simple with the stimuli used, i.e., from one to three objects in any pair of pictures, that the pre-training provided no advantage. The fact that the pre-training group (Group 1) took significantly more trials and made significantly more errors to criterion in acquiring "equal to" and "not equal to" seems puzzling. But these results may have been a function of the number of trials which the Group 1 and Group 2 Ss were taken through to acquire the inequalities concepts. As Group 1 had first to go through the simple and then the complex stimuli, each to a minimum of 8 out of 10 trials, each learner in Group 1 had a minimum of 40 trials as compared to a minimum of 20 trials for Group 2 on the inequalities apparatus. In a sense, Ss in Group 1 overlearned, acquiring at the same time a response set to respond to the stimulus containing more (or less) objects. When S reached the equipollence and non-equipollence situation, such a response set would interfere with acquisition. Every non-equipollent pair of pictures (and half of the stimuli with which he dealt with were non-equipollent) presented S with a situation in which he could respond either to the picture with more or to the picture with less objects in it, whereas the correct response would have been to press the button indicating an unequal number of objects. In support of the overlearning hypothesis, it was noticed during the experimental session that several of the children when they were learning equipollence and non-equipollence consistently responded (and

verbalized that they did so) to the pictures with more objects when presented with non-equipollent stimuli, despite repeated verbalization by E of the different concept to be learned.

Whereas it is clear that the concepts described here can be learned by any of the methods used, experimental or pedagogical, the results of Group 4 indicate that the experimental method followed by a very brief period of training with paper and pencil will be followed by virtually perfect performance. The fact that in Group 4 a small amount of training with paper and pencil after acquisition of the four concepts had been established was sufficient for the child to perform effectively on the test and that the Group 1 and Group 2 children, without any paper and pencil training, still performed as well as the pedagogically trained children who had had continuous training with paper and pencil is particularly interesting. It is often assumed in pedagogical circles that proficiency with paper and pencil should be well established before abstract concepts of the kind presented in this study are introduced. But the results described here suggest that if the concepts are firmly established (as in the experimental Groups 1 and 2), the child will transfer to a paper and pencil situation quite adequately without any previous training.

Experiment 2

In the present experiment very young children, three- and four-year-olds, learn "more than" - half with pre-training, the other half with no pre-training and performance on the final "more than" task is compared across groups. The effect of cued instructions is also examined across Ss.

There were 84 nursery school Ss ranging in age from 36 to 41 months with a mean of 38.14 months represent the three-year-olds and 48 children ranging in age from 42 to 52 months with a mean age of 46.67 months, the four-year-olds.

All the children were required to learn "more than" on the complex stimuli with from 1 to 3 objects in each picture of a pair of pictures presented on each trial. Each child was taken through 36 trials a day to a criterion of eight successive correct responses or a maximum of 108 trials. Half of the children at each age level were given pre-training trials on the simple stimuli, i.e., the pictures were all made up of striated balls. Half the children at each age level were given cued instructions (the word "more" was included in the instructions). The other half of the children at each age level were simply required to press one of the pictures presented to them. The cued instructions were:

> "Look at the pictures; look at this picture and look at that picture (E points to each picture in turn). Press the picture with more balls with your fingers (E points to the correct picture and the child subsequently presses that picture). The light went on; that was the right picture. Press the other picture; no light; it must be the wrong picture. Press the other picture, the one with more balls, again. Yes, the light went on. That is the right picture. Let's look at another picture."
>
> On the second trial, E again says, "Look at this picture, look at that picture (and points to each picture in turn). Press the one with more balls; make the light go on." If S responds incorrectly, E says, "Try the other picture".

The design described above is presented in Table 1.

Table 1

Experimental Design (3-year-old Ss)[1]

Groups	N	Stimulus Conditions		Instructions
1	8	simple	complex	cued
2	8		complex	cued
3	8	simple	complex	no verbal cue
4	8		complex	no verbal cue

[1]The design is repeated exactly for the 48 four-year-olds with 12 Ss in each group.

Results

Three-year-olds. Of the three-year-olds in Group 1, all 8 reached criterion on the pre-training task, and 7 reached criterion when transferring to the complex stimuli task. In Group 2 only three of the eight children reached criterion on the complex task without any pre-training. Both Group 1 and 2 children received the cued instructions. Of the two non-cued groups, six of the eight three-year-olds in Group 3 reached criterion on the pre-training task and five subsequently transferred successfully to the complex stimuli situation. In Group 4 only two of the eight children reached criterion on the complex stimuli with no pre-training.

Four-year-olds. In Group 1, all 12 of the four-year-olds met criterion on the pre-training task and successfully transferred to the more complex stimuli. In Group 2 only eight of the 12 four-year-olds met criterion on the complex task without any pre-training. Both Group 1 and Group 2 had cued instructions. Of the other two groups of four-year-olds with non-cued instructions, 11 of the 12 Ss in Group 3 reached criterion on pre-training task and successfully transferred to the more complex stimuli. In Group 4 seven of the 12 children reached criterion with complex stimuli with no pre-training.

Table 2 describes the number of Ss reaching criterion in each of the eight groups.

Table 2

Number of Ss Reaching Criterion
in 8 Groups of 3- and 4-Year-Old Children

	3-year-old Ss			4-year-old Ss		
Type of Instructions	Group	Simple Stimuli	Complex Stimuli	Group	Simple Stimuli	Complex Stimuli
cued	1 (N=8)	8	7	1 (N=12)	12	12
cued	2 (N=8)		3	2 (N=12)		8
non-cued	3 (N=8)	6	5	3 (N=12)	11	11
non-cued	4 (N=8)		2	4 (N=12)		7

Mean trials to criterion and mean errors between pre-training and no pre-training groups in transfer are significantly different at each age level and under both cued and non-cued conditions, with performance of the pre-training groups always the superior. Trials to criterion and errors for all eight groups are presented in Table 3 and Table 4, respectively.

Table 3

Mean Trials to Criterion for the Learners
in 8 Groups of 3- and 4-Year-Old Children

	3-year-old Ss			4-year-old Ss		
Type of Instructions	Group	Simple Stimuli	Complex Stimuli	Group	Simple Stimuli	Complex Stimuli
cued	1	33.63	30.00	1	13.5	13.67
cued	2		61.67	2		29.88
non-cued	3	25.5	16.6	3	32.91	18.36
non-cued	4		28.5	4		30.86

Table 4

Mean Errors to Criterion for the Learners
in 8 Groups of 3- and 4-Year-Old Children

	3-year-old Ss			4-year-old Ss		
Type of Instructions	Group	Simple Stimuli	Complex Stimuli	Group	Simple Stimuli	Complex Stimuli
cued	1	11.25	7.14	1	2.0	2.33
cued	2		23.0	2		9.75
non-cued	3	7.66	4.2	3	10.27	3.27
non-cued	4		11.5	4		11.14

The three-year old Ss in the cued no pre-training group (Group 2) take very many more trials to criterion and make more

errors than the non-cued no pre-training group (Group 4). But there are, of course, only 3 and 2 learners in Groups 2 and 4, respectively.

Discussion

It is clear from the present results that children as young as three and four years old can acquire the concept "more than", even when the stimuli presented to them are of the complex type described here, and that the effects of pre-training with simple stimuli facilitate subsequent transfer at both age levels.

In the case of the three-year-old, facilitation applies both to performance of those Ss who reach criterion and to the number of successful learners, i.e., in Groups 2 and 4 (no pre-training) only 3 and 2 Ss, respectively, met criterion in Task 2. The four-year-old learners, on the other hand, in the pre-training groups took fewer trials and made less errors to criterion, but the number of Ss who transferred successfully, although less, was not notably different from the no pre-training groups (Table 3).

The effects of including a verbal cue in the instructions has little effect at the four-year-old level, but for the three-year-olds the effect seems to be very considerable. Verbal assistance apparently retards learning. With so few learners in both groups, however, no final statement can be made.

Experiment 3

In Experiment 2, three- and four-year-olds successfully acquired "more than" when the stimuli presented to them consisted of 1 to 3 objects in a set of objects. In the present experiment the five-year-old child is required to learn both "more than" and "less than" with stimuli made up of pairs of pictures with 2 to 6 objects in each picture. In this case the following questions are posed: 1) Can the five-year-old successfully acquire inequalities under the experimental conditions described? 2) Are the concepts "more than" and "less than" of equal difficulty? 3) Does pre-training facilitate acquisition of the concepts? 4) What training situation, "more than" or "less than", most facilitates a subsequent reversal shift?

Subjects

The Ss, 60 nursery school children 4½ to 5½ years old, with a mean age of 58 months, were randomly assigned to four groups.

Design

Half the Ss first learned the concept "more than", the other half "less than". All Ss reversed to the alternate concept- the "more thans" to "less than"; the "less thans" to "more than". Half of the "more thans" had a "more than" pre-training task and half of the "less thans" had a "less than" pre-training task; the others had no pre-training.

Each task, "more than" or "less than", had two parts, the child first being taken to criterion on "simple" and then on "complex" stimuli. These two parts, easy to hard, constitute one task. All Ss after reaching criterion on both kinds of stimuli, reverse to the alternative concept. Again on the reversal they go to criterion on the simple stimuli and then on the complex stimuli. Half of the Ss also experienced pre-training. Table 5 presents conditions for each of the four groups.

Table 5

Design - Experiment 3

		Task 1	Task 2 - Reversal
Group 1	Pre-training	Learn "more than"	Learn "less than"
Group 2	No Pre-training		
Group 3	Pre-training	Learn "less than"	Learn "more than"
Group 4	No Pre-training		

In pre-training the stimuli used contain from 1 to 3 objects in each picture of any pair of pictures, whereas during the training proper, each picture contains from 2 to 6 objects. An example of both kinds of stimuli are presented in Figure 4.

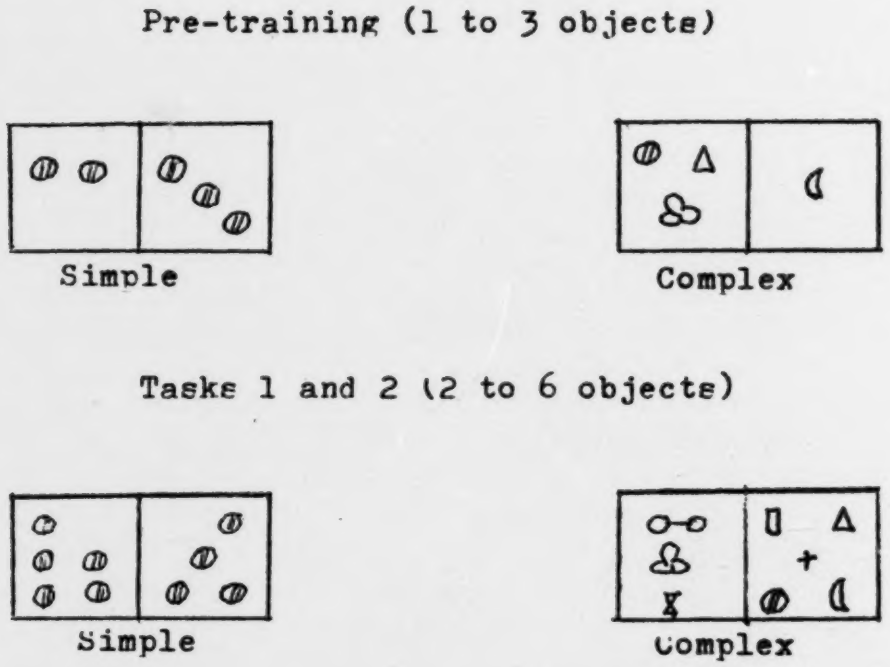

Figure 4. Examples of stimuli used in pre-training and Tasks 1 and 2.

The concepts to be learned, "more than" and "less than", were not verbalized for S who was simply told to look at each of the two pictures, press a picture, and make the light come on. The child was given 42 trials per day to a criterion of 8 out of 10 correct responses or a maximum of 126 trials on either part of the task. An overt correction procedure was used throughout.

Results

1) Fifty-eight Ss learned the pre-training and/or the pre-reversal task. Seven Ss were unable to make a reversal shift. Figures 5 and 6 show the mean number of trials to criterion for training and each task (each data point including two sets of criterion trials from the simple and complex parts of each task). The range of trials to criterion on each part of every task was from 10 to 40 trials, which includes a criterion of 10 trials. There is no doubt that children can learn the concepts and quite rapidly.

2) The question of which concept, "more than" or "less than", is easier for the young child is answered very clearly by the data presented in Figure 5. In the present experiment at least it is much easier for a five-year-old child to learn "more than". The children who initially learned "more than" and later were reversed to "less than", in fact took significantly fewer trials and made less errors than the "less than" groups across all tasks.

3) In previous experimental work by the writer with different and more difficult tasks than those presently used, pre-training not only facilitated learning, but was absolutely essential for

many children in order for them to learn. It appears here, however, that the first part of learning task 1 (simple stimuli) was sufficiently easy for the five-year-olds so that pre-training did not make a significant difference in subsequent performance.

4) Perhaps the most interesting and certainly most unexpected results of the present experiment are those which show that the children who learned on "less than" not only took longer on the initial discrimination, but also took longer to reverse to "more than" (experimentally shown to be the easier concept) than the children who reversed to the more difficult concepts, "less than". Apparently starting with "more than" was more facilitating throughout all learning in the present experimental situation.

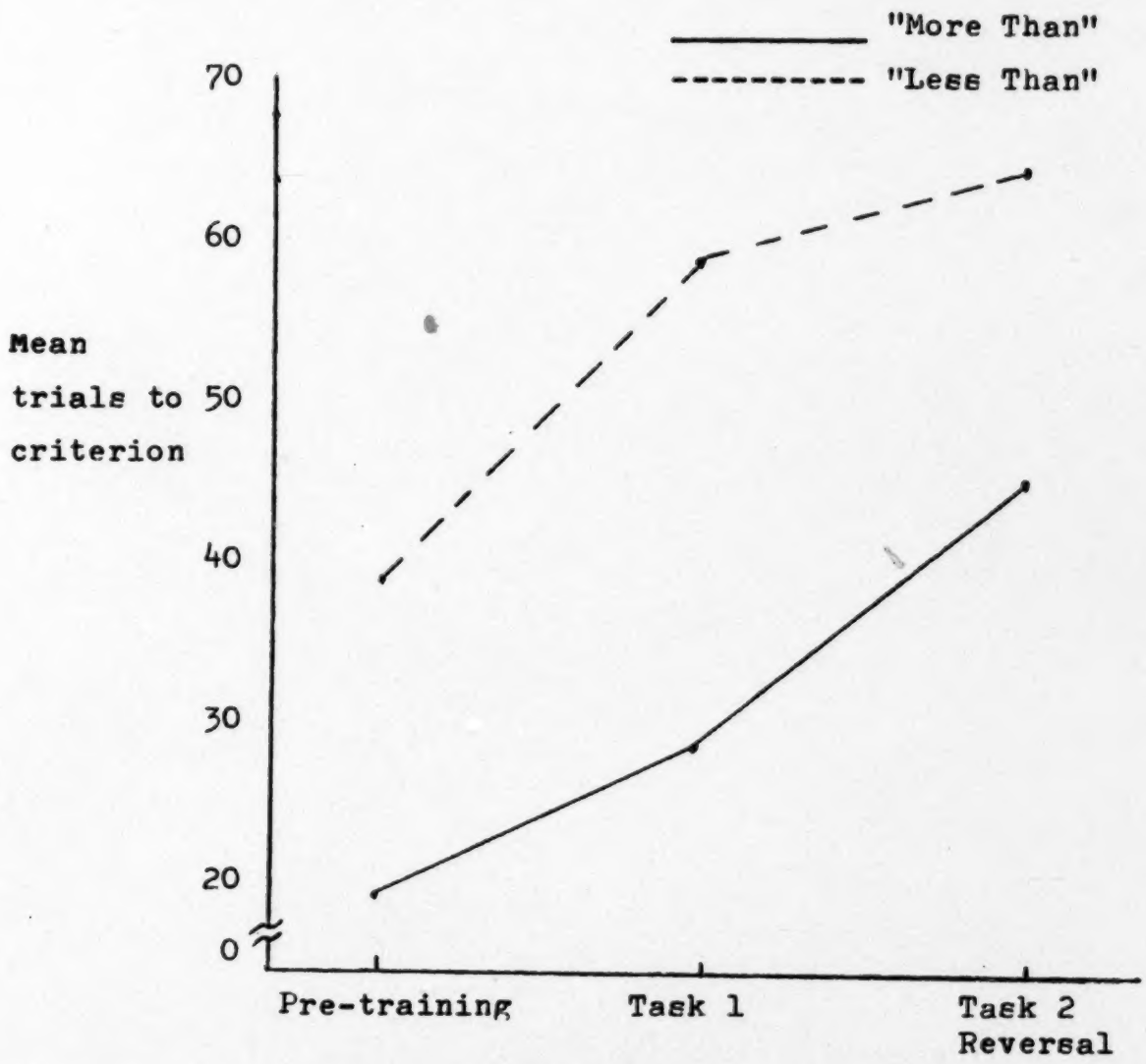

Figure 5. Mean trials to criterion on "more than" and "less than".

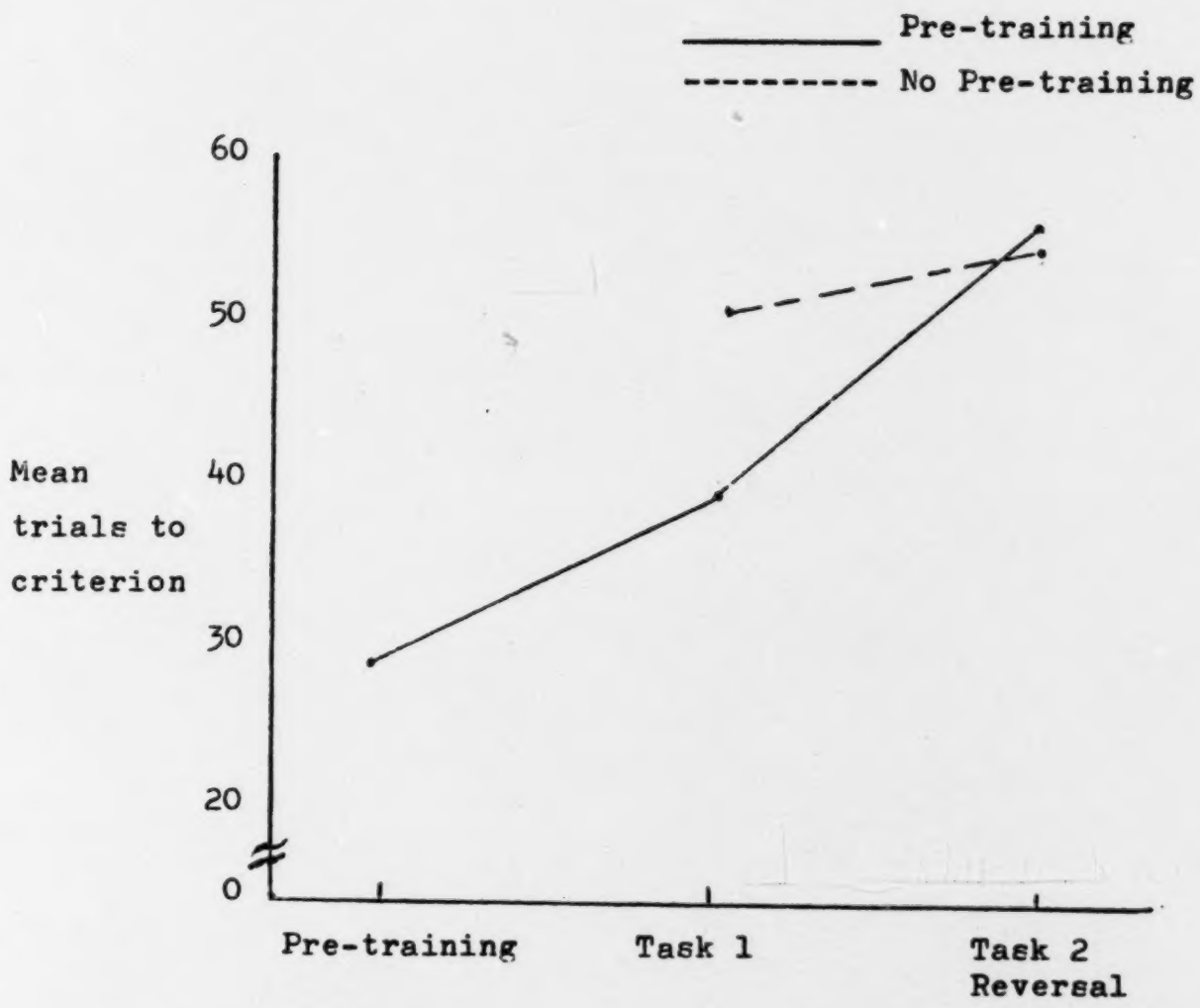

Figure 6. Mean trials to criterion by groups with pre-training and no pre-training.

Experiment 4

The five-year-old Ss of Experiment 3 learned "more than" quite easily, even with stimuli made up of pairs of pictures with from 2 to 6 objects in each picture. However in that experiment all the children were first taken to criterion on the "simple" and then transferred to "complex" stimuli. In Experiment 4 the stimuli used are again made up of from 2 to 6 objects in each set and performance of the same age children is compared between those given and those not given pre-training with the "simple" stimuli. In addition the effect of the overt correction vs. the non-correction method is examined.

Design

Sixty subjects between the ages of 4½ to 5½ years old were randomly assigned to four groups- two of which received pre-training with the simple stimuli and subsequently transferred to complex stimuli- the other two groups receiving no pre-training. The number of objects in each pair of pictures presented on any trial ranged from two to six. Half the children in the pre-training and half in the no pre-training groups were required to make an overt correction response, the other half did not. The design is presented in Table 6.

Table 6

Design - Experiment 4

Group	Response Method	Pre-training (Task 1)	Transfer (Task 2)
1	Correction	Simple stimuli	Complex stimuli
2	Non-correction	Simple stimuli	Complex stimuli
3	Correction		Complex stimuli
4	Non-correction		Complex stimuli

Results

Table 7 presents the number of learners and mean trials to criterion for each of the four groups.

Table 7

Trials to Criterion and Number of Ss Meeting Criterion in Four Groups of Five-Year-Olds ($\bar{N}$=15 in Each Group)

Group	Mean T/C Simple Stimuli	Number of Learners	Mean T/C Complex Stimuli	Number of Learners
1 Correction	20.60	15	12.73	15
2 Non-correction	37.71	14	18.92	12
3 Correction			24.36	11
4 Non-correction			38.40	10

The results presented in Table 7 are very clear. Briefly: a) The non-correction Ss (Group 2) take significantly more trials to criterion during pre-training with the "simple" stimuli than the correction group (Group 1). b) In transfer ("complex" stimuli) there is no significant difference in trials to criterion between correction and non-correction Groups 1 and 2 but the correction no pre-training Ss (Group 3) take significantly less trials to criterion than the non-correction no pre-training Ss (Group 4). c) Following pre-training Ss take significantly less trials to criterion than the no pre-training groups, both under correction and under non-correction procedures.

Discussion

In this somewhat more difficult task (2to 6 objects per set, rather than the original 1 to 3), pre-training appears to be helpful in that significantly more trials are needed by the no pre-training groups in the complex stimuli situation and at the same time 9 learners are lost in that situation as compared to 3 in the pre-training groups. Apparently also the task is sufficiently difficult to make the overt correction method facilitative at the pre-training level in Groups 1 and 2 and at the complex stimuli level in Groups 3 and 4. The number of learners however, in the present experiment, is not much affected by the response method employed.

Experiment 4 Follow-up

As a matter of interest, and as 6 of the 9 non-learners in Groups 3 and 4 (no pre-training) remained available, all 6 children were subsequently taken through the "simple" pre-training task and then, if they met criterion, transferred once more to the "complex" stimuli. Two of these six "non-learners" were from Group 4 (non-correction) and four were from Group 3 (correction). The same response procedures were maintained in the new situation.

Results. Five of the six children met criterion in pre-training and all 5 transferred successfully. There was a very large difference between the children of the original pre-training groups (28.86) and the present Ss (59.0) in mean trials to criterion on the "simple" stimuli, but the transfer means were very close (15.48 and 12.6, respectively). Apparently the pre-training session clarifies the concept very successfully for the young child - although the unsuccessful experience with the complex stimuli for our originally "non-learning" Ss may have, in consideration of the number of trials taken in pre-training, interfered considerably with subsequent performance.

Experiment 5

Experiment 5- although a follow-up of one of the experimental problems described here (cued instructions)- presents a somewhat special situation as the population cannot be considered identical with those of the previous four experiments. In the present case although the socio-economic background of the Ss is not different from that of the other groups reported- the source school is a college school used to train nursery school teachers and had a first-class teaching situation. The population is also quite stable- all the children taken through the experimental procedure had been attending the school for approximately one year- the other nursery schools from which Ss have been obtained are not often stable in this sense, children tending to come and go during the school year. Finally, and most importantly, Experiment 5 was run in the final month, May, after the children had been exposed to at least a year at a school with unusually high teaching standards. The Ss in this Experiment therefore, are initially better trained and more used to the learning situation than any of the Ss previously run.

The problem presently investigated was that of cued instructions. In Experiment 2 when three-year-olds were instructed to press the picture with "more" balls, Ss did less well than when simply told to "press a picture" - without any verbal restrictions on which picture should be chosen. In Experiment 5 the effect of verbal cues is examined with five-year-olds. In the present situation however, as it is clear from the results of Experiment 4 that the five-year-old can acquire "more than" quite easily- even with 2 to 6 objects in each set of objects presented to him- without verbal cues, the verbal cues are given only at the point at which experimental evidence indicates that the task is likely to be more difficult. The child first learns "more than" without any verbal cues and is then given reversal cues and transferred to "less than".

Twenty-eight Ss ranging from 4½ to 5½ years old were randomly assigned to four groups. Fourteen Ss received pre-training ("more than") on the simple stimuli, 14 did not. All were required to learn "more than" on the complex stimuli. Half of the pre-training and half of the no pre-training groups were given cued reversal instructions, i.e., "Now the other picture makes the light come on" (these instructions were repeated between the first few trials if the S pushed the incorrect picture). After the reversal instructions Ss were required to learn "less than" on the complex stimuli. The remaining Ss were taken through the same procedure without the cued reversal instructions. Each child was given 42 trials per day to a criterion of eight out of ten correct or a maximum of three days. Table 8 presents the Design.

Table 8

Design - Experiment 5[1]

Group	Pre-training ("more than")	Task 1 ("more than")	Task 2 Reversal ("less than")
1 (N=7)	Simple stimuli	Complex stimuli	Complex stimuli Verbal cue
2 (N=7)	Simple stimuli	Complex stimuli	Complex stimuli No cue
3 (N=7)		Complex stimuli	Complex stimuli Verbal cue
4 (N=7)		Complex stimuli	Complex stimuli No cue

[1] All stimuli are made up of from 2 to 6 objects per individual picture.

Results and Discussion

There was virtually no differences in trials to criterion between the pre-training and no pre-training Ss, but a highly significant difference between the cued and non-cued on reversal (Table 9). Only two children failed to learn all tasks, and both were lost on the reversal task, one in Group 1 and one in Group 3.

Table 9

Mean Trials to Criterion for Five-Year-Olds in Experiment 5

Group	Pre-training Simple Stimuli	Task 1 Complex Stimuli	Task 2 Reversal
1	17.57	18.14	38.50 cue
2	21.00	21.71	26.00 no cue
3		20.00	41.67 cue
4		24.29	26.00 no cue

As expected the experience of Ss in the present experiment had a profound effect upon the "pre-training vs. no pre-training" results. In Experiment 4 pre-training facilitated performance on Task 1 - in the present Experiment trials to criterion are almost identical for Task 1 across all four groups.

The results in respect to the "cue" independent variable are particularly striking in view of the comparative "sophistication" of the present Ss, confirming as they do the earlier results of Experiment 2 with the younger children. As with the three-year-olds the more explicit instructions appear to interfere with the child's learning efficiency. With no cue given (Groups 2 and 4) the children reverse to "less than" and take almost the same number of trials as in acquiring "more than" (unlike all of the earlier and less well-prepared Ss). Given a simple cue on the other hand (Groups 1 and 3) Ss take significantly more trials to acquire "less than" than they did "more than".

General Discussion and Some Conclusions

The experiments described here provide overwhelming evidence that nursery school aged children can learn simple mathematical concepts and, moreover, learn them very rapidly. An experimental trial takes no longer than 5 seconds, hence a day's session of 42 trials- including the child's introduction to the apparatus and general instructions- does not exceed 10 minutes. And a sizeable number of the children learned a concept in one session, that is, within ten minutes.

There can be little doubt that a child who recognizes six objects -all different- arranged, for example, in non-parallel lines as more than five objects -again all different- arranged in a star pattern, is responding to the number characteristic of the sets of objects. Especially so as he will on a later trial have to choose the same five objects in the same star pattern as the picture, in a pair of pictures, with the more objects. The complex stimuli, where the objects in each picture are from two to six, present even to the adult a rather complex appearance. A new experimenter in the situation is always surprised at the speed with which the child will correctly respond to the pairs of pictures presented to him on each trial.

And when, having acquired "more than", the five-year-old child reverses to "less than" his capacity to comprehend the quantitative aspects of his environment cannot be in doubt.

The comparative difficulty of "more than" and "less than" (the naive young child finds the former a much easier concept than the latter) is interesting, but perhaps not surprising in view of even the smallest child's exposure to the word "more" in his everyday life. What is, however, of particular interest is the fact that if the child first learns the easier concept "more than" and then reverses to "less than", he will take less trials to acquire the second concept than will the child who learns in the reverse order. This finding can be considered another confirmation of Lawrence's "easy to hard" paradigm, a procedure used consistently in the present experiments where the children are taken from "simple" to "complex" stimuli. Essentially one ensures that the child is presented first with absolutely clear examples which represent uniquely the concept to be learned. Once that situation is established, transfer, even to a reversal situation, seems to follow quite easily.

The pre-training condition (on "simple" stimuli) was found essential for transfer to complex stimuli, for the three-year-old child. And the five-year-old, presented with pictures containing 2 to 6 objects per picture did considerably better when first pre-trained with the simple stimuli. The age level at which pre-training is advantageous cannot, of course, be absolutely established but can be expected to change with changes in educational practice. In the years during which the present writer has performed experimental work with young children, the

child's response to number concepts appears to have accelerated considerably- probably an effect of the increased numbers of children exposed to early education, plus such outside effects as those of television's Sesame Street.

The question is whether pre-training with the simplest stimuli available, should necessarily be given. Given the experimental results presented here, plus those reported previously (Grant No. OEG-4-7-070007-2891), it seems that in the practical situation it would be entirely advantageous to include such pre-training at least for the preschool and first grade child. There is no question that with the more difficult concepts such as equipollence and non-equipollence of sets, pre-training is very important and even with the easier concepts ("more than") pre-training is essential with the very young child. But it will be noticed that in <u>all</u> the experiments reported here, more children reach criterion in the pre-training than the non-pre-training groups. Although in each experiment (except for the three-year-old group) there is no statistical difference between numbers of learners in the pre-training versus no pre-training groups; nevertheless, there are always more learners following pre-training and this goes along with fewer trials and errors to criterion. There seems no reason to omit pre-training if there is a chance that more learners may be picked up, particularly in view of the fact that at this early age even the child who learns most quickly enjoys tremendously all parts of the "game" - both simple and complex sessions. We have never, in fact, in these experiments observed boredom in the young child.

Perhaps a more pertinent question in the educational sense, is whether one should include a "complex" situation of the kind introduced in the experiments reported here. Again I would emphatically say "yes" in that transfer to a complex situation provides unarguable evidence of the child's acquisition of the concept. The ordinary kind of tests become unnecessary if the child can demonstrate such transfer- and one can detect, without giving the child any sense of failure, those who have not acquired the concept.

That the child is not given a sense of failure is considered a particular virtue of the method described here where the overt correction procedure is used. The experimental situation in fact provides a game situation for the child and with the overt correction procedure the correct picture lights up at the end of each trial- so that the child is always, finally, successful. In fact, children who have not done at all well during the session have expressed delight with their success in dealing with the task and "making the light come on". And the illusion of success does not interfere with acquisition of the concept in later sessions, as the results of Experiment 4 indicate.

In Experiment 4 the effect of the overt correction procedure was to facilitate learning with the initial "simple" stimuli-

once established there was no significant difference between correction and non-correction response groups in transfer. In the groups without pre-training the difference in mean trials and mean errors to criterion was highly significant. In previous experiments run by the present writer, with slightly older children (five and six year olds) and difficult number tasks, the same kind of facilitation of learning was found. Where the concept is very easy for the child and he acquires it immediately, requiring an overt correction response is not likely to improve performance noticeably. Nevertheless, as there is evidence of facilitation, and none whatsoever that the overt correction procedure hinders learning in any way and as it is not possible at the outset to categorize tasks into "easy" and "difficult", it would seem reasonable to include an overt correction response at least at the pre-school level. From observation the correction response appears beneficial in that it draws the child's attention to the characteristics of the correct response while at the same time, as pointed out, finishing each trial with a success experience for the child.

From the results of Experiment 1 it is reassuring to find that the pre-school child can learn at least the mathematical concepts presented here via the pedagogical method. The difficulty with the pedagogical method is, of course, that success depends very strongly indeed at this age level upon the adequacy of the teacher. The experimental method, on the other hand, depends only upon following routine steps- steps which can be performed by almost anyone given half an hour's appropriate training- teacher's aide, parent, college student. The advantage of the experimental method is probably largely that the experience given (42 trials per day to a maximum of 126 trials) is so much more for the individual child than is possible with any other method, and consequently the child is exposed to a great many different examples of the concept to be learned. The amount of experience provided for each child plus the procedural advantages- contiguity of stimulus, response and reinforcement; immediate feedback; and individual attention explains its effectiveness. This, plus ease of administration and the brevity of the period needed (10 minute sessions per child over a few days) suggest that it might provide a very powerful teaching aide in the practical situation.

From the results described here, three problems arise which deserve to be followed up to a more final conclusion. These are:

1) From the results of Experiment 1. Should paper and pencil activity with the concept to be learned be held back until the concept is firmly established? In Experiment 1 the child who first learned the four concepts required by the experimental method was able to deal quite well with the pencil and paper test without prior worksheet experience. Where the child did receive minimal training with worksheets, following acquisition via the experimental method, his performance on paper and pencil test was virtually perfect.

2) From Experiments 2 and 5. Should verbal activity with the concept to be learned be held back until the concept is firmly established? In Experiments 2 and 5 giving a verbal cue to the child "Press the picture with more balls" or "Now the other picture makes the light come on" were both found to slow down acquisition. This is so surprising a result (at least beyond the three-year-old level) that the effects of simple cueing must be re-examined most carefully.

3) Experiment 2. In the experimental situation described, does the three-year-old, in fact, learn "more than" as a number concept? It can, for example, be suggested that as the stimuli presented to the child are made up of pictures of from 1 to 3 objects only, the child may be learning to approach many objects (3) and avoid a single object. But, in the first place as each number in the final task, is made up of different objects arranged in different patterns it seems parsimonious to assume that he is at least responding to the number property of the pictures presented. And secondly, as the older children gave the same responses as the three-year-olds, when presented with similar stimuli in the first experiments, but demonstrated that they were indeed acquiring "more than" when presented with stimuli made up of two to six objects, it seems reasonable to assume that the three-year-old is learning the same concept. It is, in fact, difficult to devise situations for the three-year-old which are not too difficult for him and which will indicate unequivocally what he is learning. Perhaps the best investigation of this question would be a follow-up of the present three-year-olds where, at a later age, their success and speed in acquiring inequalities with stimuli involving at least 2 to 6 objects are compared with naive children of the same age.